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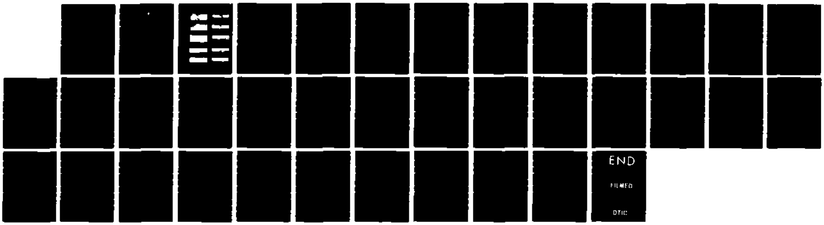

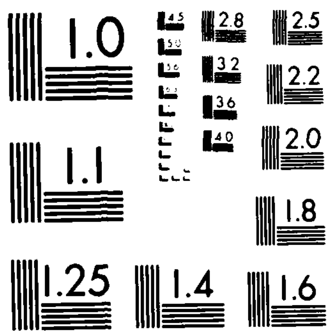

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DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

AERONAUTICAL RESEARCH LABORATORIES

MELBOURNE, VICTORIA

STRUCTURES REPORT 406

ELASTIC RESPONSE OF A HALF-PLANE TO A BONDED INTERFERENCE-FIT DISC OF THE SAME MATERIAL

by

G. S. JOST and R. P. CAREY

APPROVED FOR PUBLIC RELEASE

COPY No JULY 1984

1 Inch thick austenitic hand
welded plate Cadillac 140.
Max. size electrode: 1/4 in. dia.

1/2 Inch thick austenitic han
welded plate Cadillac 178.
Max. size electrode: 5/32 in. d

1 Inch thick austenitic hand
welded plate Cadillac 167.
Max. size electrode: 5/16 in. dia.

1/2 Inch thick austenitic hand
welded plate Gen. Motors Truck .
Max. size electrode: 3/16 in. d

1 Inch thick austenitic Unionmelt
welded plate Gen. Motors Truck 34.

1/2 Inch thick ferritic hand
welded plate Fisher H-114.
Max. size electrode: 1/4 in. dia

3/4 Inch thick austenitic Unionmelt
welded plate Fisher U 37.

1/2 Inch thick austenitic Unionm
welded plate Gen. Motors Truck 3.

3/4 Inch thick austenitic Unionmelt
welded plate Fisher U 39.

1/2 Inch thick ferritic Unionmel
welded plate Gen. Motors Truck Y

WTN.639-6677

Figure 9. Typical Weld Joint Fractures.

AR-003-940

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SUMMARY

Analytical expressions for the stress and displacement fields in a half-plane containing a bonded interference-fit disc of the same material have been enunciated. Some representative geometric configurations have been evaluated, the results being given in both tabular and graphical forms.

POSTAL ADDRESS: Director, Aeronautical Research Laboratories,
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NOMENCLATURE

a	distance from hole centre to plate boundary (free edge)
A	$= a+x = 2a+r\cos\theta$
B	$= y = r\sin\theta$
E	Young's modulus
G	Shear modulus $= E/[2(1+\nu)]$
i	$\sqrt{-1}$
P	pressure to reduce disc to radius R [$= 4G\lambda/(\kappa-1)$]
q	$= P(\kappa-1)/(\kappa+1)$
r	radius from hole centre to point z
R	radius of hole
u_x, v_y	Cartesian displacements in x and y directions
u_r, v_θ	polar displacements, radial and circumferential
x, y	Cartesian coordinates
z	location of point under consideration ($= x+iy$)
$\widehat{xx}$, $\widehat{yy}$	Cartesian stresses
$\widehat{xy}$	Cartesian shear stress
θ	angle at hole centre between x axis and point z
κ	$= (3-\nu)/(1+\nu)$ for plane stress; $(3-4\nu)$ for plane strain
λ	radial interference/R
ν	Poisson's ratio
π	*pi*
ϕ, ψ	stress functions
$\widehat{\theta\theta}$, $\widehat{rr}$	polar stresses, circumferential and radial
$\widehat{r\theta}$	polar shear stress

1. INTRODUCTION

This paper is concerned with the evaluation of the elastic stress and displacement fields arising from the bonding of an interference-fit disc in a hole near the free-edge of a large plate, Fig. 1. Such a situation is approximated quite closely in many applications where the use of interference-fit fasteners has been found highly beneficial in extending fatigue life.

The exact solutions for the title subject have been provided by Richardson[1] in the form of stress functions for both disc and plate together with two tantalizing graphs showing the influence of polar angle and geometry on the stresses. This Report presents explicitly the formulations for stress and displacement together with tabulations and graphs for several representative geometric configurations.

Both Cartesian and polar formulations are given, the derivations using the complex representation used by Muskhelishvili.[2] Non-dimensional presentations are adopted throughout.

It is to be noted that the analysis is valid only for disc and plate of the same material and where there is no failure of the bond nor relative movement around the interface.

2. STRESS FUNCTIONS

Stresses and displacements are found directly from the stress functions $\phi(z)$ and $\psi(z)$ as follows:[2]

$$\widehat{xx}+\widehat{yy} = \widehat{rr}+\widehat{\theta\theta} = 2[\phi'(z)+\overline{\phi'(z)}] = 4\,Re\,[\phi'(z)] \tag{1}$$

$$\widehat{yy}-\widehat{xx}+2i\,\widehat{xy} = 2[\bar{z}\,\phi''(z)+\psi'(z)] \tag{2}$$

$$\widehat{\theta\theta}-\widehat{rr}+2i\,\widehat{r\theta} = (\widehat{yy}-\widehat{xx}+2i\,\widehat{xy})\,e^{2i\theta} \tag{3}$$

and

$$2G(u_x+iv_y) = \kappa\phi(z)-z\overline{\phi'(z)}-\overline{\psi(z)} \tag{4}$$

$$2G(u_r+iv_\theta) = 2G(u+iv)e^{-i\theta} \tag{5}$$

The stress functions for the present problem have been derived by Richardson using the superposition principle of Eshelby[3] as follows (the notation is that of Muskhelishvili):

For the disc:

$$\Phi(z) = \frac{PR^2(\kappa-1)}{\kappa+1}\left\{-\frac{1}{2R^2}+\frac{1}{(z+a)^2}\right\}$$

$$\Psi(z) = \frac{PR^2(\kappa-1)}{\kappa+1}\left\{-\frac{1}{(z+a)^2}+\frac{2a}{(z+a)^3}\right\}$$

For the plate:

$$\Phi(z) = \frac{PR^2(\kappa-1)}{\kappa+1}\left\{\frac{1}{(z+a)^2}\right\}$$

$$\Psi(z) = \frac{PR^2(\kappa-1)}{\kappa+1}\left\{\frac{1}{(z-a)^2} - \frac{1}{(z+a)^2} + \frac{2a}{(z+a)^3}\right\}$$

For subsequent use here, the integrals of $\Phi(Z)$ and $\Psi(Z)$ are also required. Substituting the dummy variable q for $P(\kappa-1)/(\kappa+1)$ above yields the stress functions below:

For the disc:

$$\phi(z) = qR^2\left\{-\frac{z}{2R^2} - \frac{1}{z+a}\right\} \tag{6}$$

$$\psi(z) = qR^2\left\{\frac{1}{z+a} - \frac{a}{(z+a)^2}\right\} \tag{7}$$

For the plate:

$$\phi(z) = qR^2\left\{-\frac{1}{z+a}\right\} \tag{8}$$

$$\psi(z) = qR^2\left\{-\frac{1}{z-a} + \frac{1}{z+a} - \frac{a}{(z+a)^2}\right\} \tag{9}$$

These functions, together with their derivatives are now used with (1), (2) and (3) to determine disc and plate stresses, and with (4) and (5) to determine displacements.

3. STRESSES

Substitution of (6), (7), (8) and (9) into (1), (2) and (3) yield the following expressions for the stresses in disc and plate:

*For the disc** ($0 \leqq r \leqq R$):

$$\begin{pmatrix} \widehat{\theta\theta}/q \\ \widehat{rr}/q \end{pmatrix} = -1 + 2R^2\frac{A^2-B^2}{(A^2+B^2)^2} \mp \frac{LR^2}{(A^2+B^2)^3} \tag{10}$$

$$\widehat{r\theta}/q = \frac{MR^2}{(A^2+B^2)^3} \tag{11}$$

* Notice that, at the interface ($r = R$), the circumferential disc stress (10, upper) is equal to that for the plate (12, upper) minus 2: radial and shear stresses for disc and plate are identical at all points on the interface.

For the plate ($r \geqq R$):

$$\begin{pmatrix} \widehat{\theta\theta}/q \\ \widehat{rr}/q \end{pmatrix} = \pm \frac{R^2}{r^2} + 2R^2 \frac{A^2 - B^2}{(A^2+B^2)^2} \mp \frac{LR^2}{(A^2+B^2)^3} \tag{12}$$

$$\widehat{r\theta}/q = \frac{MR^2}{(A^2+B^2)^3} \tag{13}$$

Cartesian stresses for the plate are

$$\begin{pmatrix} \widehat{xx}/q \\ \widehat{yy}/q \end{pmatrix} = 2R^2 \frac{A^2 - B^2}{(A^2+B^2)^2} \mp NR^2 \tag{14}$$

$$\widehat{xy}/q = -\frac{2R^2B(A-2a)}{[(A-2a)^2+B^2]^2} + \frac{2R^2B[A(5A^2-3B^2)-2a(3A^2-B^2)]}{(A^2+B^2)^3} \tag{15}$$

where

$$\begin{pmatrix} L \\ M \end{pmatrix} = \left\{ 2[4AB(A^2-B^2) - aB(3A^2-B^2)] \begin{pmatrix} \sin 2\theta \\ \cos 2\theta \end{pmatrix} + rB(3A^2-B^2) \begin{pmatrix} \sin 3\theta \\ \cos 3\theta \end{pmatrix} \right.$$

$$\left. \pm 2[A^4 - 6A^2B^2 + B^4 - aA(A^2-3B^2)] \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix} \pm rA(A^2-3B^2) \begin{pmatrix} \cos 3\theta \\ \sin 3\theta \end{pmatrix} \right\}$$

and

$$N = \left\{ \frac{(A-2a)^2 - B^2}{[(A-2a)^2+B^2]^2} - \frac{3A^4 - 12A^2B^2 + B^4 - 4aA(A^2-3B^2)}{(A^2+B^2)^3} \right\}.$$

Some useful simplifications in these expressions occur at particular points. It is convenient to use the polar expressions for stress in the plate around the hole and Cartesian stresses along the free edge.

Around the hole $r = R$, but only inconsequential simplifications occur from this substitution. An expression for the maximum circumferential stress is impractical, as is that for the angular position at which it occurs: recourse must therefore be made to numerical solutions. Expressions for the stresses at $\theta = 0$ (on the x axis remote from the free edge) and $\theta = \pi$ (close to free edge) are, however, tractable:

$$(\widehat{rr}/q)_0^\pi = -4 \frac{2\left(\frac{a}{R}\right)^3 \mp 3\left(\frac{a}{R}\right)^2 \pm 1}{\left(2\frac{a}{R} \mp 1\right)^3}$$

$$(\widehat{\theta\theta}/q)_0^\pi = 4\left(\frac{a}{R}\right) \frac{2\left(\frac{a}{R}\right)^2 \mp 3\left(\frac{a}{R}\right) + 2}{\left(2\frac{a}{R} \mp 1\right)^3}$$

$$(\widehat{r\theta}/q)_0^\pi = 0.$$

Along the x axis $B = 0$, $A = a+x$ and (14) and (15) become

$$\widehat{xx}/q = \frac{\frac{a}{R} + 5\frac{a}{R}}{\left(\frac{a}{R} + \frac{x}{R}\right)^3} - \frac{1}{\left(\frac{a}{R} - \frac{x}{R}\right)^2}$$

$$\widehat{yy}/q = \frac{3\frac{a}{R} - \frac{x}{R}}{\left(\frac{a}{R} + \frac{x}{R}\right)^3} + \frac{1}{\left(\frac{a}{R} - \frac{x}{R}\right)^2}$$

and

$$\widehat{xy}/q = 0$$

where

$$0 \leqq x \leqq a - R$$

and

$$a + R \leqq x.$$

The minimum value of $\widehat{yy}/q$ occurs at

$$x = 0 \cdot 2232a.$$

Along the free edge $x = 0$ and (14) and (15) become

$$\widehat{xx}/q = 0$$

$$\widehat{yy}/q = 4\frac{\left(\frac{a}{R}\right)^2 - \left(\frac{y}{R}\right)^2}{\left[\left(\frac{a}{R}\right)^2 + \left(\frac{y}{R}\right)^2\right]^2}$$

and

$$\widehat{xy}/q = 0.$$

$\widehat{yy}/q$ has its maximum at the origin, where

$$\widehat{yy}/q = \frac{4}{(a/R)^2}.$$

It becomes zero when $y = \pm a$ beyond which it remains negative, reaching a minimum of

$$\widehat{yy}/q = -\frac{1}{2}\frac{1}{(a/R)^2}$$

at $y = \pm\sqrt{3}a$ and asymptoting to zero at infinity.

Plate stresses calculated from (12) to (15) are given in Tables 1 and 2 for representative values of a/R from one to infinity. This covers the full range, from a hole whose surface just touches the free edge to a hole in an infinite plate. Circumferential, radial and shear stresses around the hole are given along with Cartesian stresses along the axis of symmetry (x axis) and the free edge (y axis). Selected data are plotted in Figs 2, 3 and 4. These reveal the following:

Along the free edge both normal and shear stresses are, of course, zero. Fig. 2 shows the variation in tangential stress with distance from the origin as quantified in Table 1. The maximum non-dimensional tangential stress occurs always at the origin and has a value of 4 when $a/R = 1$, decreasing rapidly as a/R increases. The general behaviour away from the origin has already been described and may be seen readily in Fig. 2.

Around the hole and *along the axis of symmetry* the plate stresses for several values of a/R are shown in Fig. 3. Here the location of the hole is fixed, the free edge being sited progressively further to the left with increasing a/R. The scale across the hole diameter is linear in θ. The higher peak circumferential stresses clearly correspond to lower a/R, the peak non-dimensional value of 4 occurring at $a/R = 1$ where the hole and free edge meet. The maximum circumferential stress remains on the free edge at the origin for low a/R until, with increasing a/R, it falls below that at the hole. The minimum stress at the hole occurs always at the point most remote from the free edge, $\theta = 0$, from which point the circumferential stresses along the x axis diminish in a remarkably similar manner for *all* a/R.

Radial stresses behave in an understandable manner except perhaps for the small region near $\theta = \pi$ where, for very low a/R, positive values are developed: the bonding at the interface allows this as a sensible outcome. Except for the lowest a/R, shear stresses are comparatively quite small and undergo a sign change at points asymptoting from $\theta = \pi$ towards $\theta = \pi/2$ with increasing a/R: they are zero along the axis of symmetry.

The behaviour of significant non-dimensional circumferential stresses described above can be seen more clearly in Fig. 4. As a function of a/R, this shows the variation in tangential stress at the free edge at the origin, and circumferential stress at the hole nearest the origin ($\theta = \pi$),* most remote from the origin ($\theta = 0$), and in terms of its maximum value. Fig. 4 also shows how the location of this maximum changes with a/R. Up to $a/R = 1 \cdot 724$ it is fixed at the position $\theta = \pi$, but beyond this value its location moves rapidly around the hole towards its final $\pi/2$ asymptote for large a/R. The very small difference between the maximum stress around the hole and that at $\theta = \pi$ is to be noted. For a/R up to $1 \cdot 767$ the maximum circumferential stress in the plate occurs at the free edge at the origin: beyond this value it occurs at the hole. Beyond $a/R = 1 \cdot 770$ the hole stress at $\theta = \pi$ exceeds that at the origin and beyond $a/R = 1 \cdot 975$ stresses at all points around the hole exceed those at the free edge.

Contours of the stress field in the plate in terms of maximum principal stress are shown in Figs 5(a) to 5(d) for several values of a/R. These confirm the influence of a/R on both location and magnitude of maximum principal stress. The behaviour of these latter is shown more clearly in Fig. 6.

* For the disc, there is a range of a/R from 1 to $1 \cdot 261$ where the circumferential stress at the interface near the free edge is tensile, corresponding to that region where the non-dimensional plate stress exceeds 2.

4. DISPLACEMENTS

Substitution of (6), (7), (8) and (9) into (4) and (5) yield the required expressions for displacements. They are expressed here *relative to that of the centre of the disc*: i.e. relative to a stationary *disc centre.**

For the disc:

$$\frac{2G}{q}\frac{u_{rd}}{R} = \frac{(1-\kappa)r}{2R} + UR \tag{16}$$

$$\frac{2G}{q}\frac{v_{\theta d}}{R} = VR \tag{17}$$

For the plate:

$$\frac{2G}{q}\frac{u_{rp}}{R} = \frac{R}{r} + UR \tag{18}$$

$$\frac{2G}{q}\frac{v_{\theta p}}{R} = VR \tag{19}$$

In Cartesian coordinates, plate displacements are

$$\frac{2G}{q}\frac{u_x}{R} = R\left\{\frac{1+\kappa}{2a} + \frac{A-2a}{(A-2a)^2+B^2} - W_1\right\} \tag{20}$$

$$\frac{2G}{q}\frac{v_y}{R} = R\left\{\frac{B}{(a-2a)^2+B^2} - W_2\right\} \tag{21}$$

where the denominator terms $(A-2a)^2 + B^2 \equiv r^2$,

$$\begin{pmatrix} U \\ V \end{pmatrix} = \pm\frac{1+\kappa}{2a}\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} \mp \frac{(1+\kappa)A\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} \pm (1-\kappa)B\begin{pmatrix}\sin\theta \\ \cos\theta\end{pmatrix}}{A^2+B^2}$$

$$\frac{\mp[A(A^2-3B^2)-2a(A^2-B^2)]\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} \pm [2AB(A-2a)+B(A^2-B^2)]\begin{pmatrix}\sin\theta \\ \cos\theta\end{pmatrix}}{(A^2+B^2)^2}$$

$$W_1 = \frac{(1+\kappa)A}{A^2+B^2} + \frac{A(A^2-3B^2)-2a(A^2-B^2)}{(A^2+B^2)^2}$$

and

$$W_2 = \frac{(1-\kappa)B}{A^2+B^2} + \frac{B(3A^2-B^2)-4aAB}{(A^2+B^2)^2}$$

---

* The non-dimensional rigid body displacement of the plate is

$$\frac{(1+\kappa)R}{2a}$$

in the positive x direction.

At all points around the interface the sum of the magnitudes of the displacements of disc and plate must equal the radial interference. In terms of the above non-dimensional presentation, subtracting (16) from (18),

$$\frac{2G}{q}\left(\frac{u_{rp}}{R}-\frac{u_{rd}}{R}\right)=1-\frac{1-\kappa}{2}$$

$$=\frac{1+\kappa}{2}.$$

$(1+\kappa)/2$ is the non-dimensional radial interference between disc and plate.

As for the stresses, some simplifications occur at particular points. Radial displacements at the interface at $\theta=0$ and $\theta=\pi$ follow from (16) and (18), and from (20) and (21) displacements of the free edge in the x and y directions are found:

Around the interface $r=R$ and (16) and (18) give, for the disc

$$\left(\frac{2G}{q}\frac{u_{rd}}{R}\right)_0^\pi=\frac{1-\kappa}{2}+\frac{(1+\kappa)R^2}{2a(2a\mp R)}-\frac{R^2}{(2a\mp R)^2}$$

and for the plate

$$\left(\frac{2G}{q}\frac{u_{rp}}{R}\right)_0^\pi=\mp 1\mp\frac{(1+\kappa)R^2}{2a(2a\mp R)}\pm\frac{R^2}{(2a\mp R)^2}$$

Along the free edge $x=0$ and (20) and (21) become

$$\frac{2G}{q}\frac{u_x}{R}=\frac{(1+\kappa)R}{2a}-\frac{(1+\kappa)aR}{a^2+y^2}$$

and

$$\frac{2G}{q}\frac{v_y}{R}=\frac{(1+\kappa)yR}{a^2+y^2}.$$

At the origin and at infinity

$$\left(\frac{2G}{q}\frac{u_x}{R}\right)_0=-\frac{(1+\kappa)R}{2a}=-\left(\frac{2G}{q}\frac{u_x}{R}\right)_\infty$$

and

$$\left(\frac{2G}{q}\frac{v_y}{R}\right)_0=0=\left(\frac{2G}{q}\frac{v_y}{R}\right)_\infty.$$

At $y=\pm a$, $u_x=0$ and v_y has its maximum value given by

$$\frac{2G}{q}\frac{v_y}{R}=\frac{(1+\kappa)R}{2a}.$$

The appropriate equations have been used to evaluate plate displacements for representative values of a/R from 1 to infinity for plane stress and plane strain conditions along the axis of symmetry (x axis) and the free edge (y axis), Tables 3 and 4, and around the hole, Tables 5 and 6. Figures 7 and 8 show the effect of a/R on the transverse displacement of the free edge and on the radial displacement of the hole respectively. Fig. 9 shows the displacement fields for some low value a/R (a bipolar grid is used for its orthogonality to *both* boundaries). The Tables and Figures show the following:

*Along the free edge,** the transverse displacements show the expected maximum bulge at the origin for the lowest a/R, reducing to zero at $y = a$ and asymptoting towards the rigid body displacement position at infinity, Fig. 7. As previously noted, origin and remote transverse free edge displacements are of equal magnitude but opposite sign.

Around the interface, where the sum of the magnitudes of the radial displacements (i.e. the interference) is constant at all points, the maximum radial expansion of the plate occurs always at the minimum section between hole and edge: this also corresponds to the minimum compression of the disc and reference to Fig. 8 aids in understanding the existence of tensile circumferential *disc* stresses in this region at low a/R. The minimum radial expansion of the plate occurs at $\theta \sim 105^\circ$ for $a/R = 1$ and asymptotes towards $\theta = \pi/2$ with increasing a/R†: its location is therefore remarkably stable. Circumferential displacements of both disc and plate are identical at all points around the interface.

5. CONCLUSIONS

For a bonded interference-fit disc in an elastic half plane of the same material, the present analysis reveals the following:

(*a*) The maximum tangential stress along the free edge occurs always at the point nearest the hole.

(*b*) The maximum circumferential plate stress at the hole occurs at the point nearest the free edge up to an edge distance ratio of 1·724. Beyond this value the location of the maximum stress around the hole changes monotonically.

(*c*) The maximum tangential stress at the free edge exceeds the maximum circumferential stress at the hole only up to an edge distance ratio of 1·767. Beyond a ratio of 1·975 the stresses everywhere around the hole exceed the maximum stress at the free edge.

(*d*) The insertion of an interference-fit disc results in a relative displacement of disc and plate.

(*e*) Considering the disc centre as fixed, the maximum displacement of the free edge at the point nearest the hole equals the rigid body displacement of the plate in the opposite direction.

(*f*) The maximum out-of-round distortion of the interface occurs in the region nearest the free edge. It is a maximum when the hole and free edge just meet.

* All displacements for both plate and disc are with respect to a fixed *disc* centre.

† Note that the abscissa is drawn through the ordinate value unity, not zero: all radial displacements are positive, i.e. outwards.

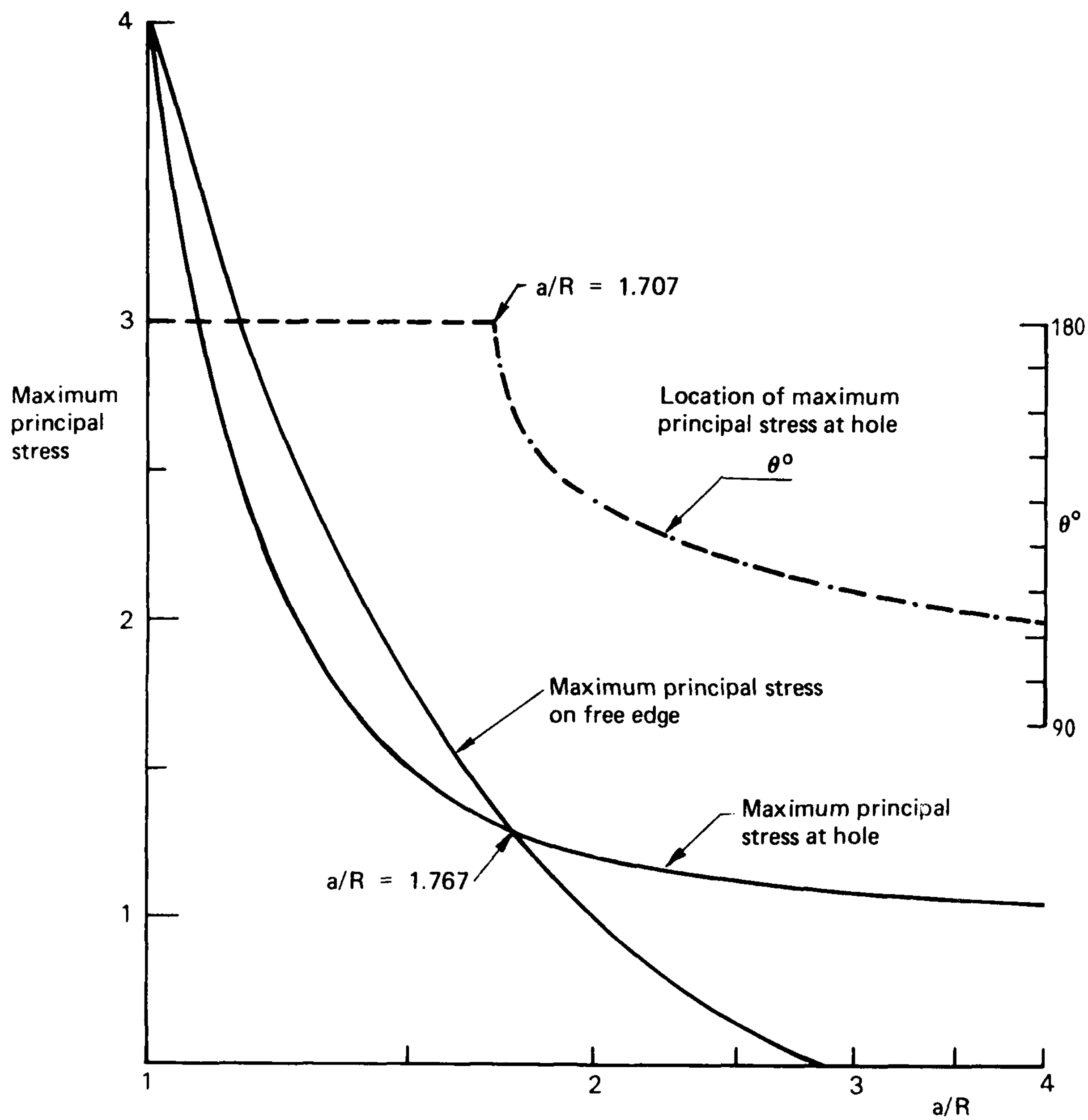
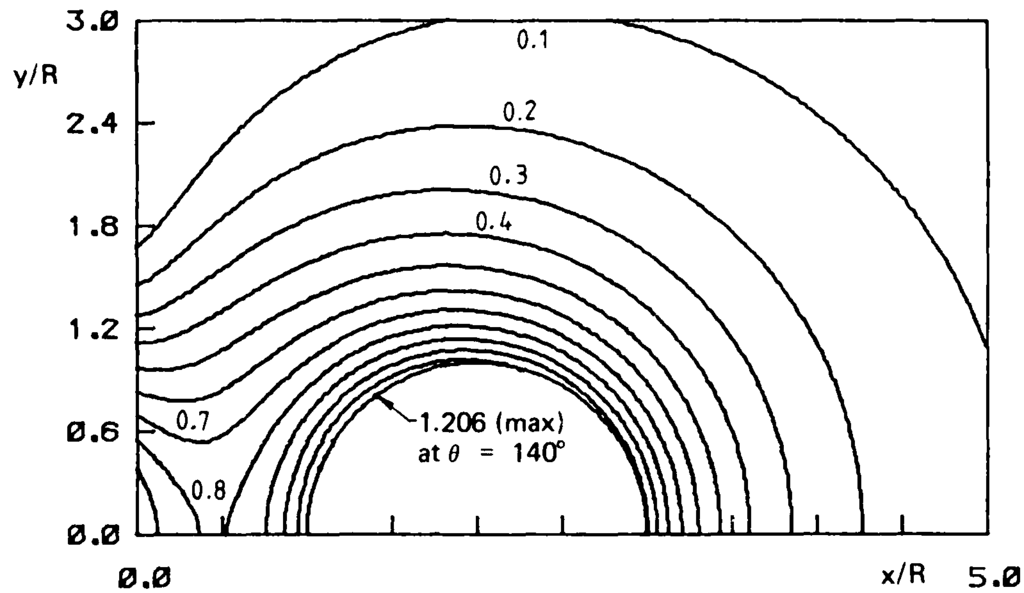
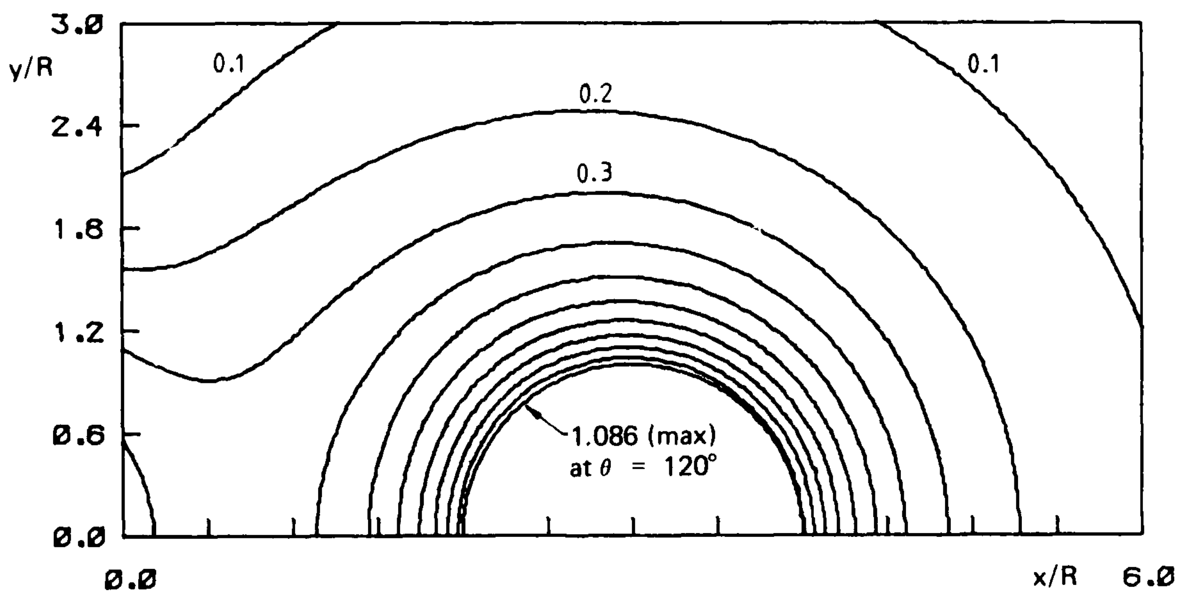


FIG. 6 MAXIMUM PRINCIPAL STRESSES AT HOLE AND FREE EDGE

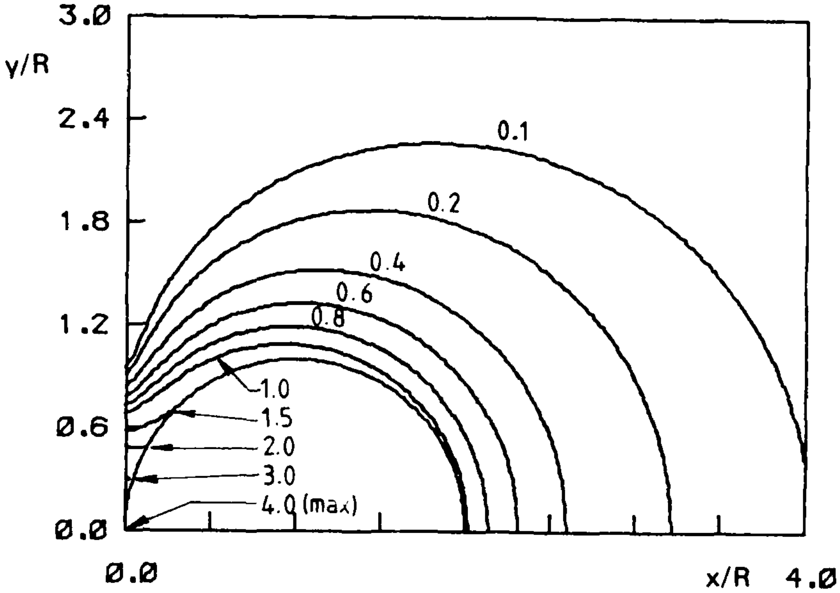


(c) a/R = 2.0 (contour interval = 0.1)

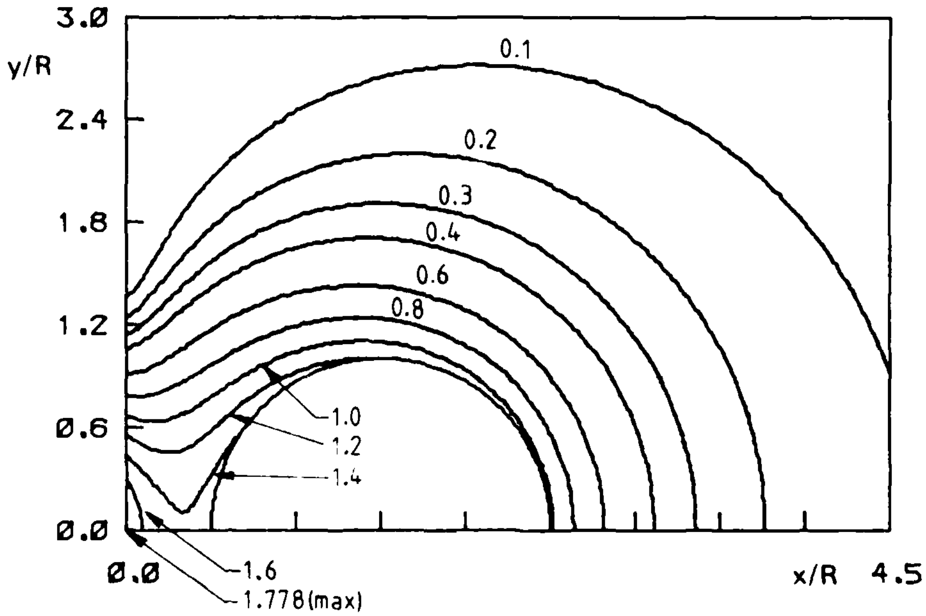


(d) a/R = 3.0 (contour interval = 0.1)

FIG. 5(c), (d) CONTOURS OF MAXIMUM NON-DIMENSIONAL PRINCIPAL STRESS



(a) a/R = 1.0



(b) a/R = 1.5

FIG. 5(a), (b) CONTOURS OF MAXIMUM NON-DIMENSIONAL PRINCIPAL STRESS

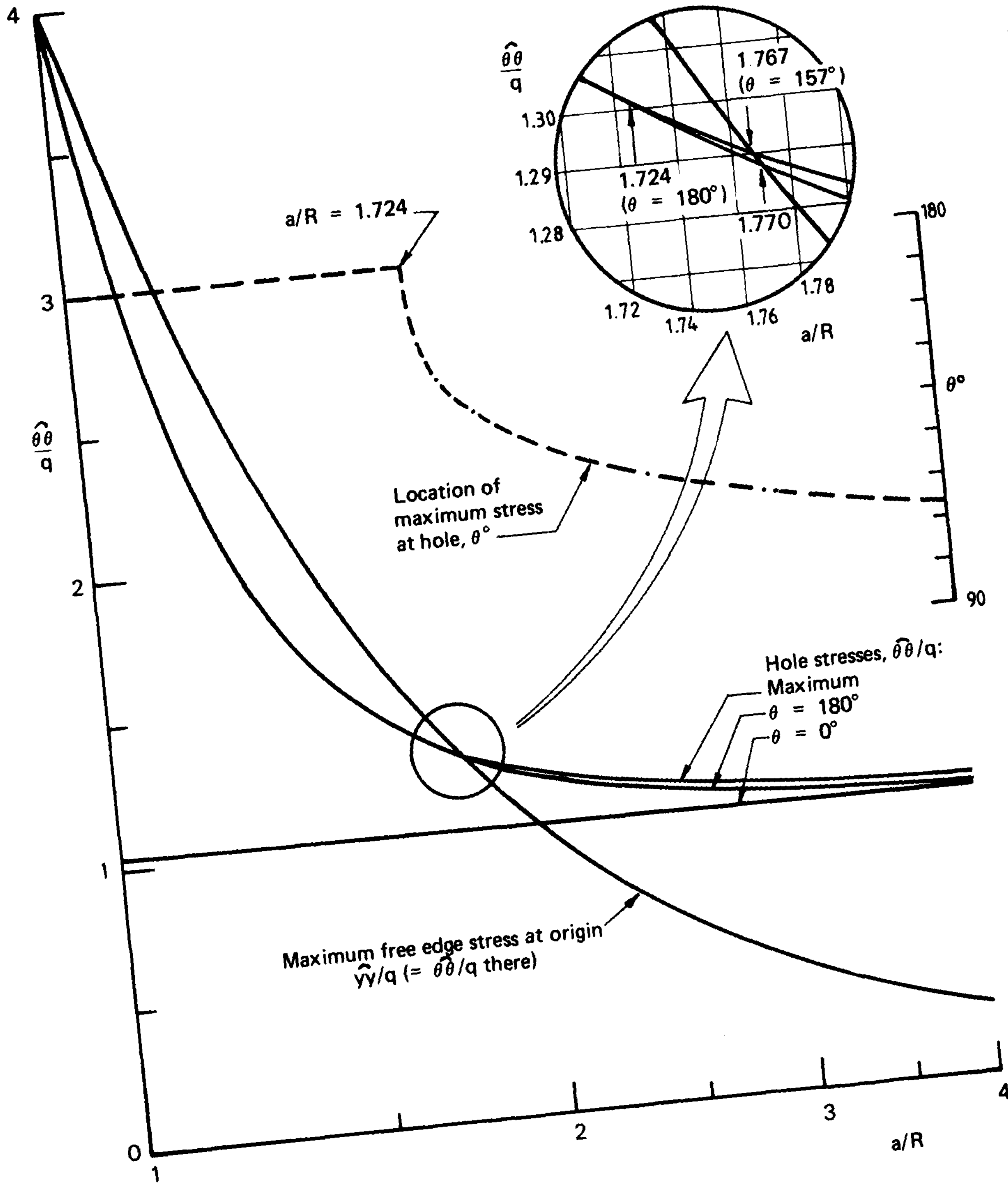


FIG. 4 INFLUENCE OF a/R ON STRESSES AT HOLE AND FREE EDGE, AND LOCATION OF MAXIMUM HOLE STRESS

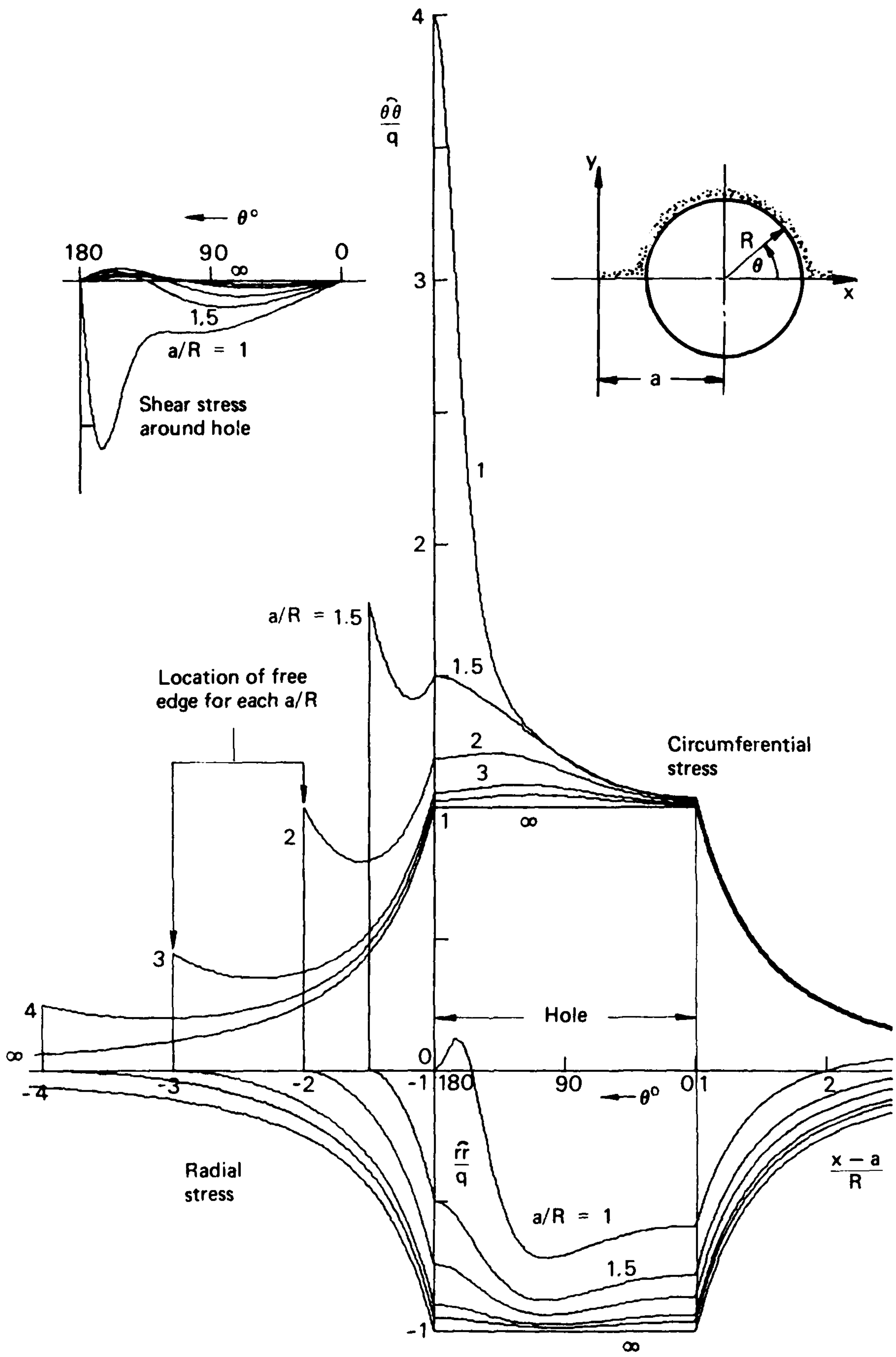


FIG. 3   STRESSES AROUND HOLE AND ALONG AXIS OF SYMMETRY

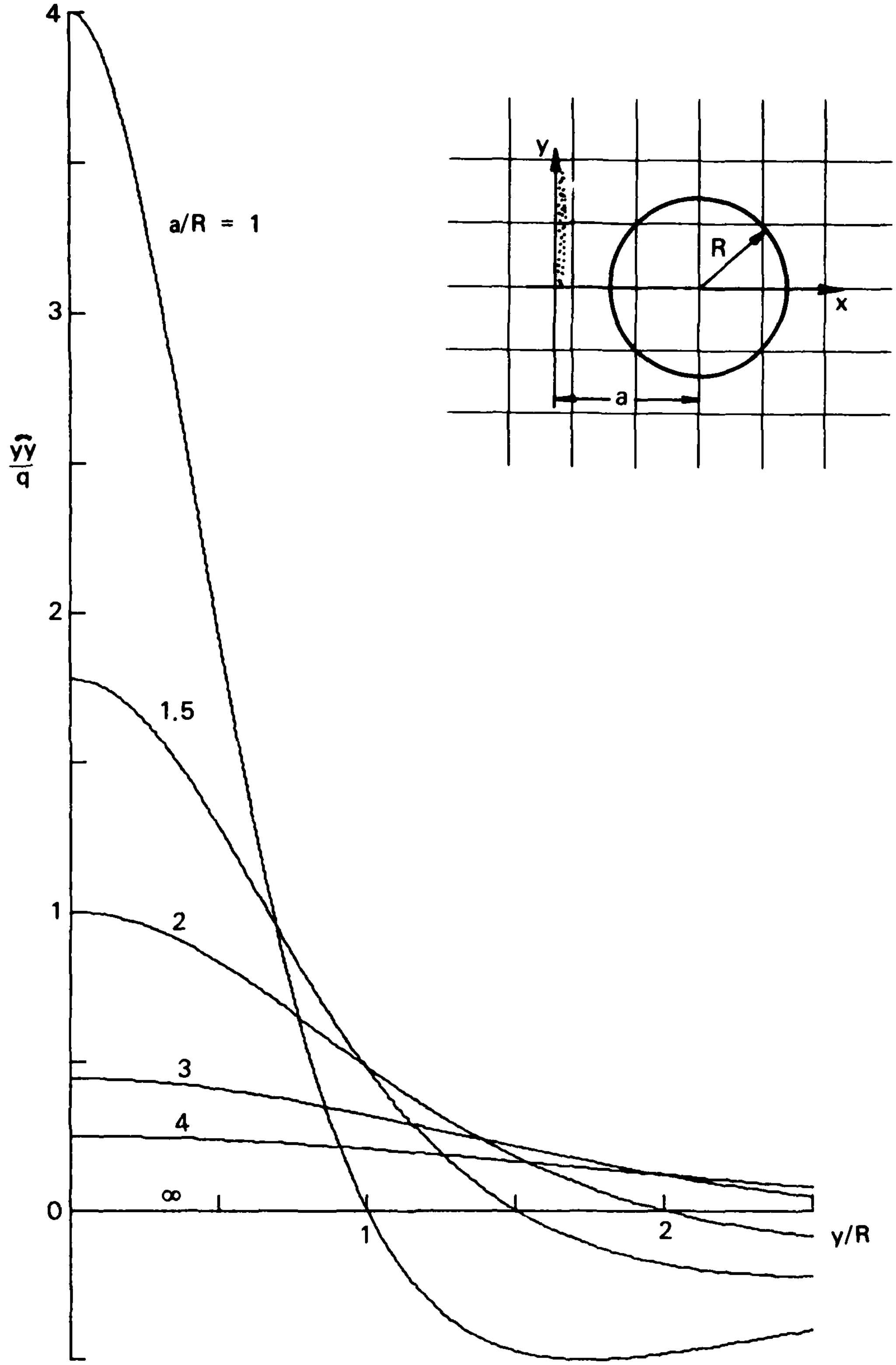


FIG. 2 TANGENTIAL STRESS ALONG FREE EDGE

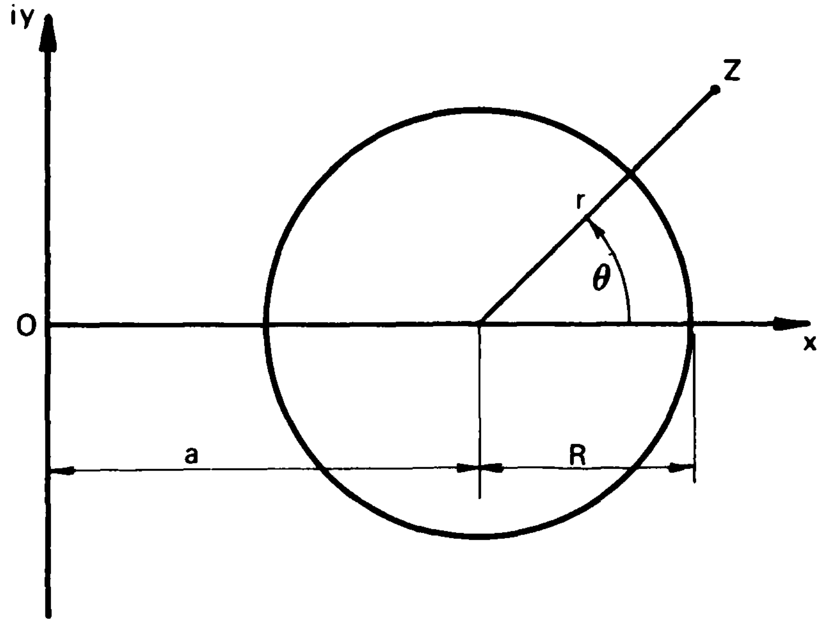


FIG. 1 CARTESIAN AND POLAR COORDINATE SYSTEM IN HALF PLANE CONTAINING HOLE

Table 6 NON-DIMENSIONAL PLATE DISPLACEMENTS - PLANE STRAIN (ν=0.3)

(a) Axis of Symmetry

a/R=1.0 x/R		a/R=1.5 x/R		a/R=2.0 x/R		a/R=3.0 x/R		a/R=4.0 x/R		a/R= ∞ (x-a)/R	
0.0	-1.4000	.00	-0.9333	0.0	-0.7000	0.0	-0.4667	0.0	-0.3500	-3.0	-0.3333
0.0	-1.4000	.05	-0.9592	.1	-0.7288	.2	-0.4920	.3	-0.3713	-2.8	-0.3571
0.0	-1.4000	.10	-0.9841	.2	-0.7564	.4	-0.5166	.6	-0.3921	-2.6	-0.3846
0.0	-1.4000	.15	-1.0085	.3	-0.7843	.6	-0.5426	.9	-0.4149	-2.4	-0.4167
0.0	-1.4000	.20	-1.0331	.4	-0.8139	.8	-0.5724	1.2	-0.4421	-2.2	-0.4545
0.0	-1.4000	.25	-1.0585	.5	-0.8467	1.0	-0.6083	1.5	-0.4764	-2.0	-0.5000
0.0	-1.4000	.30	-1.0852	.6	-0.8841	1.2	-0.6535	1.8	-0.5219	-1.8	-0.5556
0.0	-1.4000	.35	-1.1137	.7	-0.9279	1.4	-0.7121	2.1	-0.5843	-1.6	-0.6250
0.0	-1.4000	.40	-1.1447	.8	-0.9803	1.6	-0.7902	2.4	-0.6734	-1.4	-0.7143
0.0	-1.4000	.45	-1.1788	.9	-1.0438	1.8	-0.8979	2.7	-0.8082	-1.2	-0.8333
0.0	-1.4000	.50	-1.2167	1.0	-1.1222	2.0	-1.0533	3.0	-1.0296	-1.0	-1.0000
2.0	1.3556	2.50	1.1708	3.0	1.1000	4.0	1.0463	5.0	1.0265	1.0	1.0000
2.2	1.2411	2.70	1.0320	3.2	0.9505	4.2	0.8880	5.2	0.8648	1.2	0.8333
2.4	1.1696	2.90	0.9389	3.4	0.8478	4.4	0.7770	5.4	0.7506	1.4	0.7143
2.6	1.1238	3.10	0.8740	3.6	0.7740	4.6	0.6955	5.6	0.6660	1.6	0.6250
2.8	1.0941	3.30	0.8274	3.8	0.7193	4.8	0.6337	5.8	0.6011	1.8	0.5556
3.0	1.0750	3.50	0.7933	4.0	0.6778	5.0	0.5854	6.0	0.5500	2.0	0.5000
3.2	1.0632	3.70	0.7681	4.2	0.6457	5.2	0.5470	6.2	0.5089	2.2	0.4545
3.4	1.0563	3.90	0.7492	4.4	0.6206	5.4	0.5160	6.4	0.4752	2.4	0.4167
3.6	1.0530	4.10	0.7350	4.6	0.6007	5.6	0.4905	6.6	0.4473	2.6	0.3846
3.8	1.0523	4.30	0.7245	4.8	0.5848	5.8	0.4695	6.8	0.4239	2.8	0.3571
4.0	1.0533	4.50	0.7167	5.0	0.5721	6.0	0.4519	7.0	0.4040	3.0	0.3333
∞	1.4000		0.9333		0.7000		0.4667		0.3500		0.0000

(b) Free Edge

	a/R	1.0	1.5	2.0	3.0	4.0
y/R	0.0	-1.4000	-0.9333	-0.7000	-0.4667	-0.3500
	.2	-1.2923	-0.9007	-0.6861	-0.4625	-0.3483
	.4	-1.0138	-0.8094	-0.6462	-0.4504	-0.3431
	.6	-0.6588	-0.6759	-0.5844	-0.4308	-0.3346
	.8	-0.3073	-0.5200	-0.5069	-0.4047	-0.3231
	1.0	0.0000	-0.3590	-0.4200	-0.3733	-0.3088
	1.2	0.2525	-0.2049	-0.3294	-0.3379	-0.2922
	1.4	0.4541	-0.0643	-0.2396	-0.2998	-0.2736
	1.6	0.6135	0.0602	-0.1537	-0.2600	-0.2534
	1.8	0.7396	0.1683	-0.0735	-0.2196	-0.2321
	2.0	0.8400	0.2613	0.0000	-0.1795	-0.2100
	∞	1.4000	0.9333	0.7000	0.4667	0.3500

Table 5 NON-DIMENSIONAL PLATE DISPLACEMENTS - PLANE STRESS (ν=0.3)

(a) Axis of Symmetry

a/R=1.0		a/R=1.5		a/R=2.0		a/R=3.0		a/R=4.0		a/R= ∞	
x/R		x/R		x/R		x/R		x/R		(x-a)/R	
0.0	-1.5385	.00	-1.0256	0.0	-0.7692	0.0	-0.5128	0.0	-0.3846	-3.0	-0.3333
0.0	-1.5385	.05	-1.0456	.1	-0.7914	.2	-0.5324	.3	-0.4011	-2.8	-0.3571
0.0	-1.5385	.10	-1.0648	.2	-0.8130	.4	-0.5519	.6	-0.4177	-2.6	-0.3846
0.0	-1.5385	.15	-1.0840	.3	-0.8354	.6	-0.5734	.9	-0.4368	-2.4	-0.4167
0.0	-1.5385	.20	-1.1037	.4	-0.8600	.8	-0.5991	1.2	-0.4607	-2.2	-0.4545
0.0	-1.5385	.25	-1.1244	.5	-0.8882	1.0	-0.6314	1.5	-0.4922	-2.0	-0.5000
0.0	-1.5385	.30	-1.1467	.6	-0.9214	1.2	-0.6733	1.8	-0.5350	-1.8	-0.5556
0.0	-1.5385	.35	-1.1711	.7	-0.9613	1.4	-0.7288	2.1	-0.5951	-1.6	-0.6250
0.0	-1.5385	.40	-1.1982	.8	-1.0099	1.6	-0.8042	2.4	-0.6821	-1.4	-0.7143
0.0	-1.5385	.45	-1.2285	.9	-1.0701	1.8	-0.9095	2.7	-0.8149	-1.2	-0.8333
0.0	-1.5385	.50	-1.2628	1.0	-1.1453	2.0	-1.0626	3.0	-1.0345	-1.0	-1.0000
2.0	1.4017	2.50	1.1939	3.0	1.1138	4.0	1.0529	5.0	1.0304	1.0	1.0000
2.2	1.2931	2.70	1.0583	3.2	0.9665	4.2	0.8957	5.2	0.8693	1.2	0.8333
2.4	1.2267	2.90	0.9683	3.4	0.8657	4.4	0.7857	5.4	0.7557	1.4	0.7143
2.6	1.1853	3.10	0.9061	3.6	0.7938	4.6	0.7053	5.6	0.6717	1.6	0.6250
2.8	1.1596	3.30	0.8620	3.8	0.7408	4.8	0.6443	5.8	0.6075	1.8	0.5556
3.0	1.1442	3.50	0.8303	4.0	0.7009	5.0	0.5970	6.0	0.5569	2.0	0.5000
3.2	1.1357	3.70	0.8071	4.2	0.6703	5.2	0.5594	6.2	0.5164	2.2	0.4545
3.4	1.1319	3.90	0.7902	4.4	0.6465	5.4	0.5292	6.4	0.4832	2.4	0.4167
3.6	1.1313	4.10	0.7779	4.6	0.6280	5.6	0.5045	6.6	0.4558	2.6	0.3846
3.8	1.1331	4.30	0.7690	4.8	0.6133	5.8	0.4842	6.8	0.4329	2.8	0.3571
4.0	1.1364	4.50	0.7628	5.0	0.6018	6.0	0.4672	7.0	0.4134	3.0	0.3333
∞	1.5385		1.0256		0.7692		0.5128		0.3846		0.0000

(b) Free Edge

	a/R	1.0	1.5	2.0	3.0	4.0
y/R	0.0	-1.5385	-1.0256	-0.7692	-0.5128	-0.3846
	.2	-1.4201	-0.9898	-0.7540	-0.5083	-0.3827
	.4	-1.1141	-0.8895	-0.7101	-0.4949	-0.3770
	.6	-0.7240	-0.7427	-0.6422	-0.4734	-0.3677
	.8	-0.3377	-0.5714	-0.5570	-0.4447	-0.3550
	1.0	0.0000	-0.3945	-0.4615	-0.4103	-0.3394
	1.2	0.2774	-0.2251	-0.3620	-0.3714	-0.3211
	1.4	0.4990	-0.0706	-0.2633	-0.3294	-0.3007
	1.6	0.6742	0.0661	-0.1689	-0.2857	-0.2785
	1.8	0.8128	0.1850	-0.0807	-0.2413	-0.2551
	2.0	0.9231	0.2872	0.0000	-0.1972	-0.2308
	∞	1.5385	1.0256	0.7692	0.5128	0.3846

Table 4 NON-DIMENSIONAL POLAR DISPLACEMENTS AROUND INTERFACE FOR PLATE AND DISC-PLANE STRAIN (ν=0.3)

a/R=1.0

ANGLE	PLATE RADIAL	CIRCUM	DISC RADIAL
0	1.3556	0.0000	-0.0444
10	1.3496	-0.0784	-0.0504
20	1.3320	-0.1543	-0.0680
30	1.3038	-0.2250	-0.0962
40	1.2664	-0.2884	-0.1336
50	1.2221	-0.3424	-0.1779
60	1.1735	-0.3853	-0.2265
70	1.1241	-0.4161	-0.2759
80	1.0780	-0.4342	-0.3220
90	1.0400	-0.4400	-0.3600
100	1.0157	-0.4349	-0.3843
110	1.0114	-0.4215	-0.3886
120	1.0333	-0.4041	-0.3667
130	1.0863	-0.3886	-0.3137
140	1.1695	-0.3795	-0.2305
150	1.2690	-0.3726	-0.1310
160	1.3537	-0.3387	-0.0463
170	1.3939	-0.2201	-0.0061
180	1.4000	0.0000	0.0000

a/R=1.5

ANGLE	PLATE RADIAL	CIRCUM	DISC RADIAL
0	1.1708	0.0000	-0.2292
10	1.1674	-0.0372	-0.2326
20	1.1572	-0.0725	-0.2428
30	1.1409	-0.1044	-0.2591
40	1.1196	-0.1313	-0.2804
50	1.0946	-0.1517	-0.3054
60	1.0679	-0.1647	-0.3321
70	1.0415	-0.1693	-0.3585
80	1.0179	-0.1655	-0.3821
90	1.0000	-0.1533	-0.4000
100	0.9905	-0.1338	-0.4095
110	0.9921	-0.1086	-0.4079
120	1.0068	-0.0801	-0.3932
130	1.0354	-0.0518	-0.3646
140	1.0762	-0.0271	-0.3238
150	1.1240	-0.0093	-0.2760
160	1.1703	0.0001	-0.2297
170	1.2042	0.0022	-0.1958
180	1.2167	0.0000	-0.1833

a/R=2.0

ANGLE	PLATE RADIAL	CIRCUM	DISC RADIAL
0	1.1000	0.0000	-0.3000
10	1.0977	-0.0214	-0.3023
20	1.0911	-0.0416	-0.3089
30	1.0805	-0.0593	-0.3195
40	1.0668	-0.0735	-0.3332
50	1.0510	-0.0832	-0.3490
60	1.0344	-0.0878	-0.3656
70	1.0184	-0.0868	-0.3816
80	1.0047	-0.0804	-0.3953
90	0.9952	-0.0689	-0.4048
100	0.9912	-0.0532	-0.4088
110	0.9943	-0.0348	-0.4057
120	1.0050	-0.0159	-0.3950
130	1.0232	0.0012	-0.3768
140	1.0472	0.0140	-0.3528
150	1.0738	0.0203	-0.3262
160	1.0984	0.0193	-0.3016
170	1.1159	0.0116	-0.2841
180	1.1222	0.0000	-0.2778

a/R=3.0

ANGLE	PLATE RADIAL	CIRCUM	DISC RADIAL
0	1.0463	0.0000	-0.3537
10	1.0451	-0.0097	-0.3549
20	1.0416	-0.0187	-0.3584
30	1.0362	-0.0263	-0.3638
40	1.0292	-0.0319	-0.3708
50	1.0213	-0.0351	-0.3787
60	1.0132	-0.0356	-0.3868
70	1.0058	-0.0334	-0.3942
80	0.9998	-0.0285	-0.4002
90	0.9961	-0.0214	-0.4039
100	0.9953	-0.0128	-0.4047
110	0.9978	-0.0036	-0.4022
120	1.0037	0.0050	-0.3963
130	1.0125	0.0119	-0.3875
140	1.0232	0.0159	-0.3768
150	1.0344	0.0165	-0.3656
160	1.0442	0.0136	-0.3558
170	1.0509	0.0076	-0.3491
180	1.0533	0.0000	-0.3467

a/R=4.0

ANGLE	PLATE RADIAL	CIRCUM	DISC RADIAL
0	1.0265	0.0000	-0.3735
10	1.0258	-0.0055	-0.3742
20	1.0237	-0.0105	-0.3763
30	1.0204	-0.0146	-0.3796
40	1.0162	-0.0175	-0.3838
50	1.0115	-0.0190	-0.3885
60	1.0068	-0.0188	-0.3932
70	1.0025	-0.0169	-0.3975
80	0.9993	-0.0136	-0.4007
90	0.9974	-0.0092	-0.4026
100	0.9973	-0.0040	-0.4027
110	0.9991	0.0012	-0.4009
120	1.0027	0.0059	-0.3973
130	1.0077	0.0093	-0.3923
140	1.0137	0.0111	-0.3863
150	1.0197	0.0108	-0.3803
160	1.0249	0.0085	-0.3751
170	1.0284	0.0047	-0.3716
180	1.0296	0.0000	-0.3704

a/R= ∞

ANGLE	PLATE RADIAL	CIRCUM	DISC RADIAL
0	1.0000	0.0000	-0.4000
10	1.0000	-0.0000	-0.4000
20	1.0000	-0.0000	-0.4000
30	1.0000	-0.0000	-0.4000
40	1.0000	-0.0000	-0.4000
50	1.0000	-0.0000	-0.4000
60	1.0000	-0.0000	-0.4000
70	1.0000	-0.0000	-0.4000
80	1.0000	-0.0000	-0.4000
90	1.0000	-0.0000	-0.4000
100	1.0000	0.0000	-0.4000
110	1.0000	0.0000	-0.4000
120	1.0000	0.0000	-0.4000
130	1.0000	0.0000	-0.4000
140	1.0000	0.0000	-0.4000
150	1.0000	0.0000	-0.4000
160	1.0000	0.0000	-0.4000
170	1.0000	0.0000	-0.4000
180	1.0000	0.0000	-0.4000

Table 3 NON-DIMENSIONAL POLAR DISPLACEMENTS AROUND INTERFACE FOR PLATE AND DISC-PLANE STRESS (ν=0.3)

a/R=1.0

ANGLE	PLATE RADIAL	CIRCUM	DISC RADIAL
0	1.4017	0.0000	-0.1368
10	1.3958	-0.0811	-0.1426
20	1.3785	-0.1597	-0.1599
30	1.3507	-0.2332	-0.1878
40	1.3139	-0.2994	-0.2245
50	1.2704	-0.3564	-0.2681
60	1.2229	-0.4024	-0.3155
70	1.1750	-0.4365	-0.3635
80	1.1308	-0.4581	-0.4076
90	1.0954	-0.4677	-0.4431
100	1.0745	-0.4665	-0.4640
110	1.0746	-0.4573	-0.4639
120	1.1026	-0.4441	-0.4359
130	1.1637	-0.4322	-0.3748
140	1.2577	-0.4254	-0.2807
150	1.3712	-0.4177	-0.1673
160	1.4719	-0.3768	-0.0665
170	1.5264	-0.2428	-0.0121
180	1.5385	0.0000	0.0000

a/R=1.5

ANGLE	PLATE RADIAL	CIRCUM	DISC RADIAL
0	1.1939	0.0000	-0.3446
10	1.1905	-0.0382	-0.3480
20	1.1804	-0.0746	-0.3580
30	1.1644	-0.1075	-0.3741
40	1.1434	-0.1354	-0.3951
50	1.1189	-0.1568	-0.4196
60	1.0927	-0.1708	-0.4458
70	1.0671	-0.1765	-0.4714
80	1.0445	-0.1737	-0.4940
90	1.0277	-0.1626	-0.5108
100	1.0196	-0.1439	-0.5189
110	1.0229	-0.1195	-0.5155
120	1.0398	-0.0915	-0.4987
130	1.0708	-0.0633	-0.4676
140	1.1143	-0.0381	-0.4241
150	1.1650	-0.0189	-0.3734
160	1.2139	-0.0071	-0.3246
170	1.2496	-0.0017	-0.2888
180	1.2628	0.0000	-0.2756

a/R=2.0

ANGLE	PLATE RADIAL	CIRCUM	DISC RADIAL
0	1.1138	0.0000	-0.4246
10	1.1116	-0.0219	-0.4269
20	1.1050	-0.0426	-0.4334
30	1.0946	-0.0608	-0.4438
40	1.0811	-0.0754	-0.4574
50	1.0655	-0.0856	-0.4729
60	1.0492	-0.0906	-0.4893
70	1.0336	-0.0901	-0.5048
80	1.0205	-0.0841	-0.5180
90	1.0114	-0.0729	-0.5270
100	1.0082	-0.0575	-0.5303
110	1.0120	-0.0394	-0.5264
120	1.0237	-0.0205	-0.5148
130	1.0428	-0.0032	-0.4956
140	1.0678	0.0099	-0.4706
150	1.0954	0.0169	-0.4431
160	1.1207	0.0168	-0.4177
170	1.1388	0.0103	-0.3997
180	1.1453	0.0000	-0.3932

a/R=3.0

ANGLE	PLATE RADIAL	CIRCUM	DISC RADIAL
0	1.0529	0.0000	-0.4856
10	1.0517	-0.0098	-0.4868
20	1.0483	-0.0190	-0.4902
30	1.0429	-0.0267	-0.4956
40	1.0360	-0.0325	-0.5025
50	1.0282	-0.0359	-0.5103
60	1.0202	-0.0366	-0.5183
70	1.0129	-0.0344	-0.5256
80	1.0071	-0.0296	-0.5314
90	1.0035	-0.0226	-0.5349
100	1.0030	-0.0141	-0.5355
110	1.0058	-0.0049	-0.5327
120	1.0119	0.0037	-0.5266
130	1.0210	0.0106	-0.5175
140	1.0319	0.0149	-0.5065
150	1.0433	0.0157	-0.4952
160	1.0533	0.0129	-0.4852
170	1.0601	0.0073	-0.4783
180	1.0626	0.0000	-0.4759

a/R=4.0

ANGLE	PLATE RADIAL	CIRCUM	DISC RADIAL
0	1.0304	0.0000	-0.5081
10	1.0297	-0.0055	-0.5088
20	1.0276	-0.0106	-0.5109
30	1.0243	-0.0148	-0.5142
40	1.0201	-0.0178	-0.5183
50	1.0155	-0.0193	-0.5230
60	1.0108	-0.0192	-0.5276
70	1.0066	-0.0174	-0.5318
80	1.0034	-0.0141	-0.5350
90	1.0017	-0.0097	-0.5368
100	1.0016	-0.0046	-0.5368
110	1.0035	0.0007	-0.5350
120	1.0072	0.0054	-0.5313
130	1.0124	0.0088	-0.5261
140	1.0184	0.0106	-0.5200
150	1.0245	0.0104	-0.5139
160	1.0297	0.0083	-0.5087
170	1.0333	0.0046	-0.5052
180	1.0345	0.0000	-0.5039

a/R= ∞

ANGLE	PLATE RADIAL	CIRCUM	DISC RADIAL
0	1.0000	0.0000	-0.5385
10	1.0000	-0.0000	-0.5385
20	1.0000	-0.0000	-0.5385
30	1.0000	-0.0000	-0.5385
40	1.0000	-0.0000	-0.5385
50	1.0000	-0.0000	-0.5385
60	1.0000	-0.0000	-0.5385
70	1.0000	-0.0000	-0.5385
80	1.0000	-0.0000	-0.5385
90	1.0000	-0.0000	-0.5385
100	1.0000	0.0000	-0.5385
110	1.0000	0.0000	-0.5385
120	1.0000	0.0000	-0.5385
130	1.0000	0.0000	-0.5385
140	1.0000	0.0000	-0.5385
150	1.0000	0.0000	-0.5385
160	1.0000	0.0000	-0.5385
170	1.0000	0.0000	-0.5385
180	1.0000	0.0000	-0.5385

Table 2 NON-DIMENSIONAL PLATE STRESSES AROUND HOLE

a/R=1.0

ANGLE	CIRCUM	RADIAL	SHEAR
0	1.0370	-0.5926	0.0000
10	1.0385	-0.5941	-0.0258
20	1.0431	-0.5986	-0.0515
30	1.0509	-0.6062	-0.0767
40	1.0621	-0.6169	-0.1013
50	1.0771	-0.6306	-0.1247
60	1.0962	-0.6472	-0.1464
70	1.1200	-0.6661	-0.1657
80	1.1491	-0.6860	-0.1813
90	1.1840	-0.7040	-0.1920
100	1.2253	-0.7148	-0.1966
110	1.2740	-0.7082	-0.1953
120	1.3333	-0.6667	-0.1925
130	1.4150	-0.5639	-0.2030
140	1.5581	-0.3739	-0.2609
150	1.8676	-0.1111	-0.4094
160	2.5169	0.0983	-0.6068
170	3.4702	0.0863	-0.5565
180	4.0000	0.0000	0.0000

a/R=1.5

ANGLE	CIRCUM	RADIAL	SHEAR
0	1.0313	-0.7813	0.0000
10	1.0330	-0.7826	-0.0177
20	1.0384	-0.7865	-0.0349
30	1.0475	-0.7930	-0.0514
40	1.0604	-0.8019	-0.0665
50	1.0774	-0.8132	-0.0797
60	1.0985	-0.8264	-0.0903
70	1.1240	-0.8408	-0.0972
80	1.1539	-0.8553	-0.0995
90	1.1880	-0.8680	-0.0960
100	1.2257	-0.8759	-0.0855
110	1.2661	-0.8746	-0.0676
120	1.3076	-0.8586	-0.0429
130	1.3487	-0.8219	-0.0144
140	1.3883	-0.7612	0.0123
150	1.4262	-0.6802	0.0298
160	1.4615	-0.5936	0.0323
170	1.4891	-0.5258	0.0204
180	1.5000	-0.5000	0.0000

a/R=2.0

ANGLE	CIRCUM	RADIAL	SHEAR
0	1.0240	-0.8640	0.0000
10	1.0255	-0.8651	-0.0122
20	1.0299	-0.8683	-0.0240
30	1.0373	-0.8737	-0.0349
40	1.0477	-0.8809	-0.0443
50	1.0609	-0.8898	-0.0518
60	1.0768	-0.8999	-0.0567
70	1.0951	-0.9106	-0.0582
80	1.1153	-0.9207	-0.0558
90	1.1366	-0.9290	-0.0488
100	1.1577	-0.9333	-0.0370
110	1.1770	-0.9313	-0.0205
120	1.1925	-0.9203	-0.0008
130	1.2024	-0.8985	0.0195
140	1.2055	-0.8655	0.0362
150	1.2022	-0.8248	0.0446
160	1.1950	-0.7836	0.0407
170	1.1880	-0.7524	0.0244
180	1.1852	-0.7407	0.0000

a/R=3.0

ANGLE	CIRCUM	RADIAL	SHEAR
0	1.0146	-0.9329	0.0000
10	1.0155	-0.9337	-0.0066
20	1.0183	-0.9358	-0.0129
30	1.0228	-0.9393	-0.0184
40	1.0290	-0.9439	-0.0228
50	1.0365	-0.9494	-0.0257
60	1.0452	-0.9554	-0.0268
70	1.0545	-0.9613	-0.0257
80	1.0638	-0.9665	-0.0223
90	1.0725	-0.9702	-0.0166
100	1.0796	-0.9714	-0.0088
110	1.0844	-0.9693	0.0005
120	1.0860	-0.9632	0.0101
130	1.0841	-0.9530	0.0185
140	1.0789	-0.9393	0.0239
150	1.0716	-0.9241	0.0250
160	1.0640	-0.9099	0.0207
170	1.0582	-0.8997	0.0118
180	1.0560	-0.8960	0.0000

a/R=4.0

ANGLE	CIRCUM	RADIAL	SHEAR
0	1.0096	-0.9602	0.0000
10	1.0102	-0.9607	-0.0041
20	1.0120	-0.9622	-0.0079
30	1.0150	-0.9646	-0.0112
40	1.0189	-0.9677	-0.0136
50	1.0236	-0.9713	-0.0150
60	1.0288	-0.9751	-0.0152
70	1.0341	-0.9788	-0.0139
80	1.0391	-0.9818	-0.0113
90	1.0433	-0.9837	-0.0073
100	1.0463	-0.9840	-0.0024
110	1.0476	-0.9824	0.0030
120	1.0470	-0.9787	0.0082
130	1.0444	-0.9729	0.0124
140	1.0404	-0.9657	0.0147
150	1.0355	-0.9580	0.0145
160	1.0308	-0.9511	0.0116
170	1.0275	-0.9463	0.0065
180	1.0262	-0.9446	0.0000

a/R= ∞

ANGLE	CIRCUM	RADIAL	SHEAR
0	1.0000	-1.0000	0.0000
10	1.0000	-1.0000	-0.0000
20	1.0000	-1.0000	-0.0000
30	1.0000	-1.0000	-0.0000
40	1.0000	-1.0000	-0.0000
50	1.0000	-1.0000	-0.0000
60	1.0000	-1.0000	-0.0000
70	1.0000	-1.0000	-0.0000
80	1.0000	-1.0000	-0.0000
90	1.0000	-1.0000	-0.0000
100	1.0000	-1.0000	0.0000
110	1.0000	-1.0000	0.0000
120	1.0000	-1.0000	0.0000
130	1.0000	-1.0000	0.0000
140	1.0000	-1.0000	0.0000
150	1.0000	-1.0000	0.0000
160	1.0000	-1.0000	0.0000
170	1.0000	-1.0000	0.0000
180	1.0000	-1.0000	0.0000

Table 1 NON-DIMENSIONAL PLATE STRESSES-AXIS OF SYMMETRY AND FREE EDGE

(a) Axis of Symmetry

a/R=1.0			a/R=1.5			a/R=2.0		
x/R	PERP	PARALLEL	x/R	PERP	PARALLEL	x/R	PERP	PARALLEL
0.0	4.0000	0.0000	.00	1.7778	-0.0000	0.0	1.0000	0.0000
0.0	4.0000	0.0000	.05	1.6706	-0.0057	.1	0.9141	-0.0071
0.0	4.0000	0.0000	.10	1.5844	-0.0219	.2	0.8533	-0.0269
0.0	4.0000	0.0000	.15	1.5171	-0.0478	.3	0.8145	-0.0584
0.0	4.0000	0.0000	.20	1.4669	-0.0829	.4	0.7957	-0.1013
0.0	4.0000	0.0000	.25	1.4330	-0.1269		0.7964	-0.1564
0.0	4.0000	0.0000	.30	1.4146	-0.1800	.6	0.8174	-0.2257
0.0	4.0000	0.0000	.35	1.4116	-0.2428	.7	0.8610	-0.3123
0.0	4.0000	0.0000	.40	1.4242	-0.3162	.8	0.9313	-0.4211
0.0	4.0000	0.0000	.45	1.4532	-0.4013	.9	1.0356	-0.5599
0.0	4.0000	0.0000	.50	1.5000	-0.5000	1.0	1.1852	-0.7407
2.0	1.0370	-0.5926	2.50	1.0313	-0.7813	3.0	1.0240	-0.8640
2.2	0.7189	-0.3282	2.70	0.7187	-0.4920	3.2	0.7144	-0.5664
2.4	0.5255	-0.1794	2.90	0.5290	-0.3224	3.4	0.5267	-0.3895
2.6	0.3992	-0.0906	3.10	0.4050	-0.2160	3.6	0.4043	-0.2767
2.8	0.3123	-0.0353	3.30	0.3195	-0.1459	3.8	0.3199	-0.2010
3.0	0.2500	0.0000	3.50	0.2580	-0.0980	4.0	0.2593	-0.1481
3.2	0.2039	0.0228	3.70	0.2123	-0.0644	4.2	0.2142	-0.1101
3.4	0.1689	0.0377	3.90	0.1774	-0.0402	4.4	0.1797	-0.0821
3.6	0.1418	0.0473	4.10	0.1502	-0.0227	4.6	0.1528	-0.0610
3.8	0.1203	0.0533	4.30	0.1286	-0.0097	4.8	0.1314	-0.0449
4.0	0.1031	0.0569	4.50	0.1111	-0.0000	5.0	0.1140	-0.0324

a/R=3.0			a/R=4.0			a/R= ∞		
x/R	PERP	PARALLEL	x/R	PERP	PARALLEL	(x-a)/R	PERP	PARALLEL
0.0	0.4444	-0.0000	0.0	0.2500	0.0000	-3.0	0.1111	-0.1111
.2	0.3961	-0.0055	0.3	0.2202	-0.0039	-2.8	0.1275	-0.1275
.4	0.3667	-0.0207	0.6	0.2036	-0.0146	-2.6	0.1479	-0.1479
.6	0.3537	-0.0450	0.9	0.1984	-0.0318	-2.4	0.1736	-0.1736
.8	0.3561	-0.0790	1.2	0.2044	-0.0564	-2.2	0.2066	-0.2066
1.0	0.3750	-0.1250	1.5	0.2231	-0.0909	-2.0	0.2500	-0.2500
1.2	0.4139	-0.1872	1.8	0.2589	-0.1400	-1.8	0.3086	-0.3086
1.4	0.4798	-0.2732	2.1	0.3206	-0.2131	-1.6	0.3906	-0.3906
1.6	0.5862	-0.3972	2.4	0.4272	-0.3296	-1.4	0.5102	-0.5102
1.8	0.7595	-0.5859	2.7	0.6226	-0.5335	-1.2	0.6945	-0.6945
2.0	1.0560	-0.8960	3.0	1.0262	-0.9446	-1.0	1.0000	-1.0000
4.0	1.0146	-0.9329	5.0	1.0096	-0.9602	1.0	1.0000	-1.0000
4.2	0.7073	-0.6301	5.2	0.7032	-0.6559	1.2	0.6945	-0.6945
4.4	0.5216	-0.4485	5.4	0.5182	-0.4729	1.4	0.5102	-0.5102
4.6	0.4006	-0.3314	5.6	0.3979	-0.3545	1.6	0.3906	-0.3906
4.8	0.3175	-0.2517	5.8	0.3152	-0.2736	1.8	0.3086	-0.3086
5.0	0.2578	-0.1953	6.0	0.2560	-0.2160	2.0	0.2500	-0.2500
5.2	0.2135	-0.1540	6.2	0.2121	-0.1736	2.2	0.2066	-0.2066
5.4	0.1797	-0.1230	6.4	0.1786	-0.1416	2.4	0.1736	-0.1736
5.6	0.1533	-0.0992	6.6	0.1525	-0.1169	2.6	0.1479	-0.1479
5.8	0.1322	-0.0806	6.8	0.1317	-0.0974	2.8	0.1275	-0.1275
6.0	0.1152	-0.0658	7.0	0.1149	-0.0818	3.0	0.1111	-0.1111

(b) Free Edge

a/R	1.0	1.5	2.0	3.0	4.0
y/R 0.0	4.0000	1.7778	1.0000	0.4444	0.2500
.2	3.5503	1.6857	0.9705	0.4386	0.2481
.4	2.4970	1.4394	0.8876	0.4214	0.2426
.6	1.3841	1.1098	0.7659	0.3945	0.2337
.8	0.5354	0.7711	0.6243	0.3598	0.2219
1.0	0.0000	0.4734	0.4800	0.3200	0.2076
1.2	-0.2956	0.2380	0.3460	0.2774	0.1915
1.4	-0.4383	0.0654	0.2297	0.2344	0.1741
1.6	-0.4924	-0.0536	0.1338	0.1928	0.1561
1.8	-0.4984	-0.1314	0.0580	0.1538	0.1379
2.0	-0.4800	-0.1792	0.0000	0.1183	0.1200

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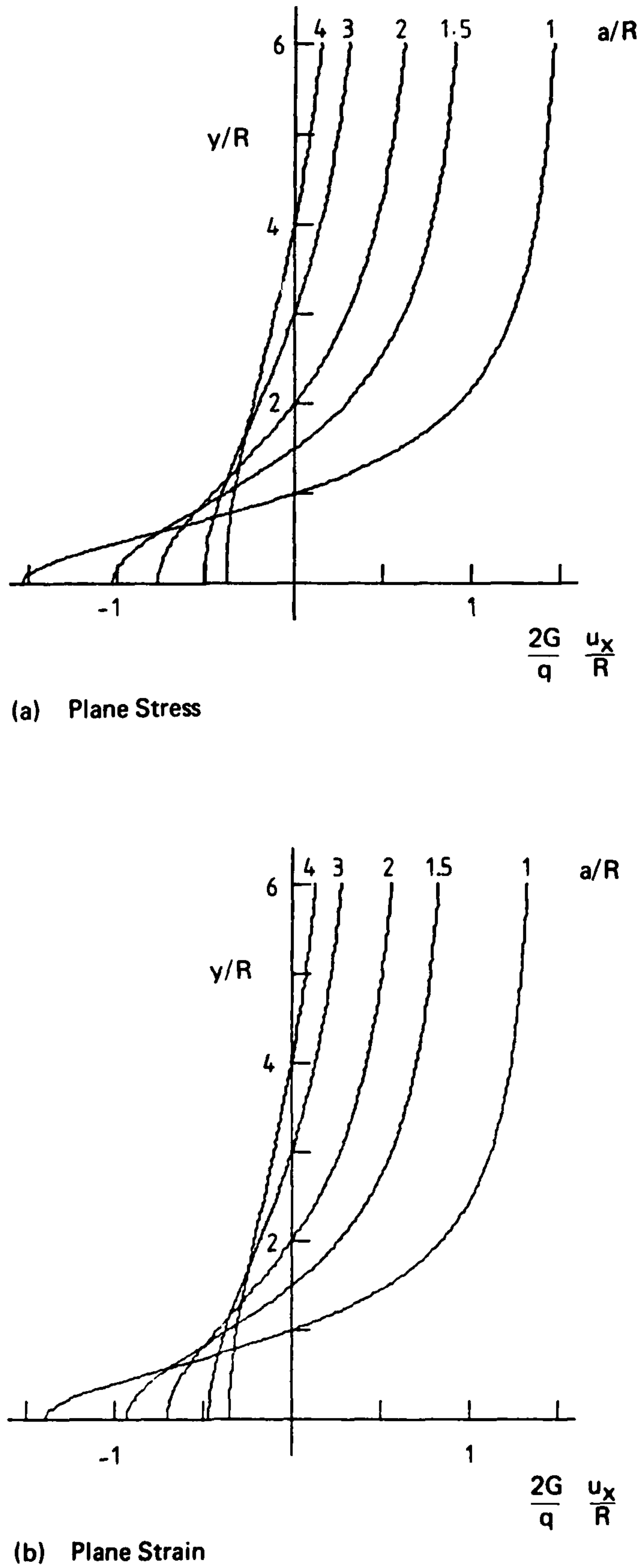
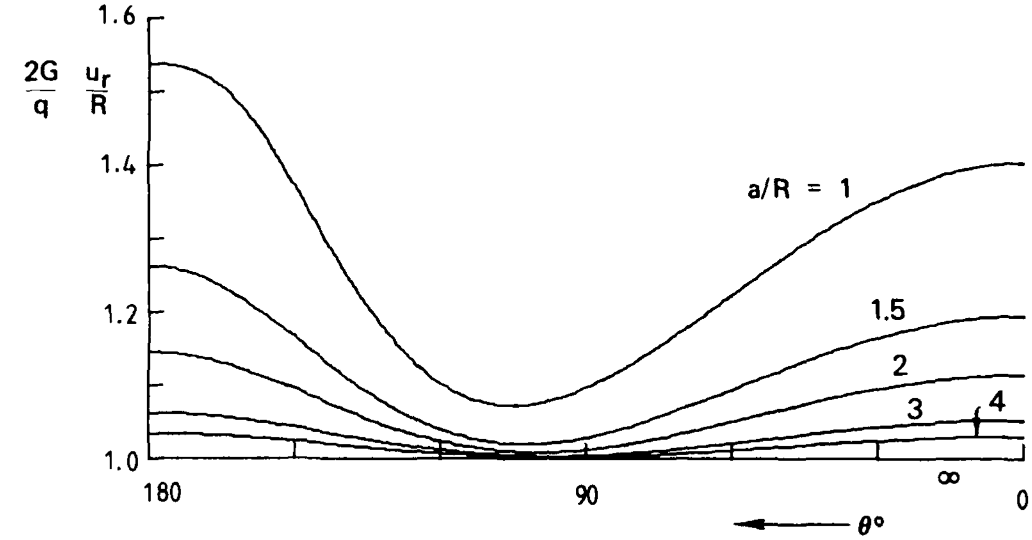
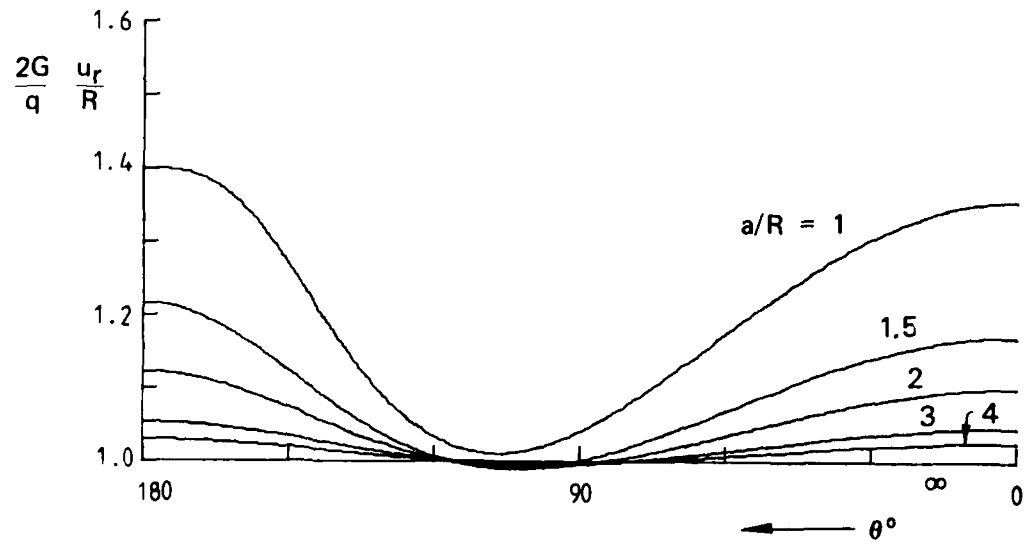


FIG. 7 TRANSVERSE DISPLACEMENT OF FREE EDGE (ν = 0.3)

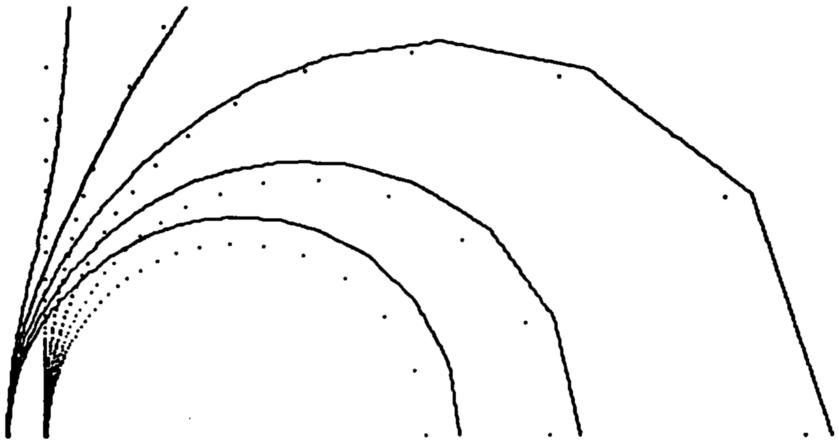


(a) Plane Stress

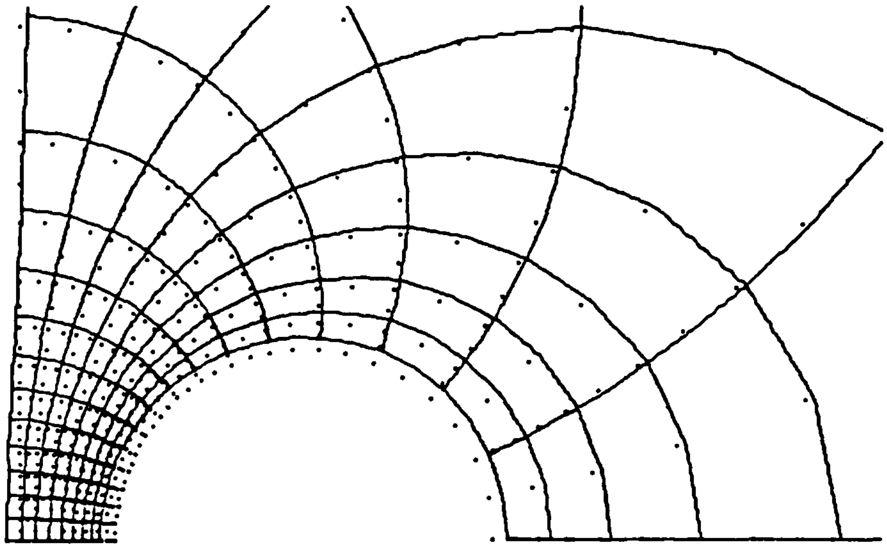


(b) Plane Strain

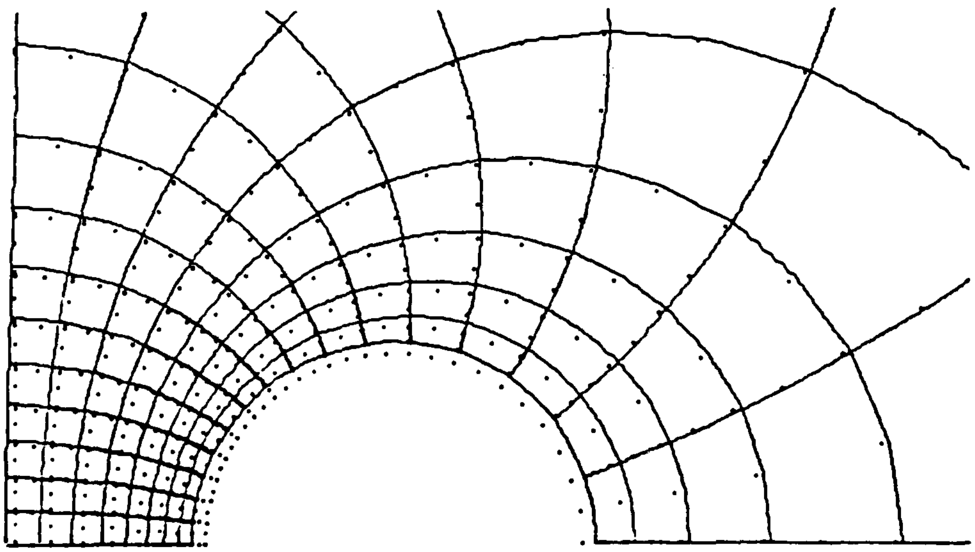
FIG. 8 RADIAL DISPLACEMENT OF HOLE (ν = 0.3)



(a) a/R = 1.0 (λ = 0.2)



(b) a/R = 1.5 (λ = 0.1)



(c) a/R = 2.0 (λ = 0.1)

FIG. 9 DEFORMATION PLOTS – PLANE STRESS (ν = 0.3)

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16. Abstract

Analytical expressions for the stress and displacement fields in a half-plane containing a bonded interference-fit disc of the same material, have been enunciated. Some representative geometric configurations have been evaluated, the results being given in both tabular and graphical forms.

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